

Joint Spectrum and Power Auction With Multiauctioneer and Multibidder in Coded Cooperative Cognitive Radio Networks

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Abstract—In the existing cooperative cognitive radio (CR), qualities of service (e.g., rate or outage probability) of primary users can be improved by cooperative transmission from secondary users. However, as owners of the spectrum, primary users' traffic demands are relatively easy to satisfy. The rationale is that they would be more interested in the benefits of other formats (e.g., revenue), rather than the enhanced rate. In this paper, we propose a new cooperative CR framework, where primary users assist in the transmissions of secondary users. In exchange for this concession, primary users receive payments from secondary users for the spectrum and cooperative transmit power being used in cooperation. An auction-theoretic model with multiple auctioneers, multiple bidders, and multiple commodities is developed for a joint spectrum and cooperative power allocation. Since the spectrum and power are two heterogeneous but correlated commodities, their properties are specifically considered in auction strategy design. Finally, we mathematically prove the convergence of the proposed auction game, and show with numerical results, that the proposed auction is beneficial to both primary and secondary users.

Index Terms—Cooperative cognitive radio, spectrum and power allocation, auction, Walrasian equilibrium.

I. INTRODUCTION

WITH the rapid deployment of wireless services in the last decade, radio spectrum is becoming a valuable and scarce resource. How to support the growing applications with limited spectrum emerges a critical issue for future wireless communications. On the other side, the report from the Federal Communications Commissions reveals that most of the licensed spectrum is severely underutilized [1]. As a promising technique, cognitive radio (CR) [2], [3] is proposed to deal with

the dilemma between spectrum scarcity and spectrum underutilization. CR allows unlicensed users (or secondary users) to access licensed bands under the condition that the induced interference to licensed users (or primary users) does not reach an unacceptable level.

Cooperative communications [4]–[6], a new technique for mitigating path loss and channel fading, has attracted much attention. It enables users to relay data for each other and thus creates a virtual multiple-input-multiple-output (MIMO) system for cooperative diversity. Recently, incorporation of cooperation concept into CR networks has become a new cognitive radio paradigm. Cooperation in CR networks is mainly classified into two categories: i) cooperation among secondary users; ii) cooperation between primary users and secondary users. The first category aims to improve the performance of secondary transmission, in which a secondary user acts as a relay and assists transmissions of other secondary users [7], [8]. Generally, the solutions for traditional cooperative communications are valid for cooperation among secondary users. The only difference is that dynamic spectrum access must be considered in the latter.

The second category benefits both primary and secondary users in which different rights of primary users and secondary users to the spectrum are taken into account, thus is more challenging than the first category. Simeone *et al.* in [9] proposed a cooperation-based spectrum leasing scheme, where a primary user leases the owned spectrum to a subset of secondary users for a fraction of time in exchange for cooperation from secondary users. Zhang *et al.* in [10] proposed a cooperative cognitive radio network framework, in which some secondary users are selected by primary users as cooperative relays and in return, they obtain more spectrum access opportunities. An asymmetric cooperative communications architecture was proposed in [11], where the secondary user relays the primary data with its own transmit power for the usage of the spectrum, while the primary user transmits only its own data. Similar works can also be found in [12]–[15].

The existing works on cooperation between primary and secondary users fall into just one direction, i.e., secondary users relay the traffic for primary users, from which primary users achieve the improvement of quality of service (e.g., transmission rate or outage probability). Actually, as owners of the spectrum, primary users' traffic demands are relatively easy to be satisfied, especially for those who have a light traffic load. To these users, the enhanced rate or the cooperation from secondary users might have less attractions. Instead, they

Manuscript received October 14, 2013; revised February 28, 2014 and May 28, 2014; accepted July 10, 2014. Date of publication July 23, 2014; date of current version October 8, 2014. The work was supported in part by the NSFC under Grants 61271211, 61228101, and U1201255. The associate editor coordinating the review of this paper and approving it for publication was Z. Han.

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Digital Object Identifier 10.1109/TWC.2014.2342211

would be more interested in the benefits of other format (e.g. revenue). In this paper, we propose a new cooperation way between primary and secondary users, where primary users assist transmissions of secondary users. In exchange for this concession, primary users receive the payments of secondary users for the spectrum and cooperative transmit power being used in cooperation.

Network coding [16] was first proposed to achieve the capacity gain of wired networks. The broadcast nature of wireless medium enables a relay to overhear messages from multiple sources, which facilitates the application of network coding in cooperative communications (also referred to as network-coded cooperation in [17]). Some network-coded cooperation schemes have been proposed in [18]–[20]. In this study, we consider a network-coded cooperative CR environment, in which primary users are endowed with network coding capability. Hence, they do not forward secondary users' data directly, but perform a combination of those with its own data (see Section II for details). In this way, the additional diversity gain can be achieved for both primary and secondary users.

In the proposed cooperative transmission, secondary users require to compete at primary users for channel bands and cooperative transmit power. Both dynamic spectrum access and power allocation in CR networks can be conducted in a centralized manner, for example, by linear programming or convex optimization algorithm. In practice, their solutions often require global information and coordination among all users, which is very costly and sometimes infeasible in distributed settings.

Auction theory [21] is a simple and powerful tool for distributed resource allocation in interactive multiuser systems. Auction-based power and channel allocation have been extensively studied in the literature [22]–[26]. For instance, the SNR auction and the power auction schemes were proposed in [22] to coordinate the relay power allocation among users on the basis of amplify-and-forward relaying protocol. The authors in [23] proposed a two-level buyer/seller game for cooperative transmit power allocation, in which the interests of source nodes and relay nodes are jointly considered. The authors in [25] formulated channel allocation problem in CR networks as an auction game and discussed three kinds of auction algorithms. A multi-auctioneer progressive auction algorithm was proposed in [26], where multiple secondary users bid for the channels from multiple primary users.

The existing power or spectrum auction models are characterized by single auctioneer multiple bidders for single commodity as in [25], or multiple auctioneers multiple bidders for single commodity, for example, for power in [24] and for channel in [26]. In order to jointly address spectrum and power allocation problem in cooperative CR networks, we propose a new auction structure with multiple auctioneers, multiple bidders and multiple commodities. Primary users, acting as auctioneers, sell a portion of the channel access time and cooperative transmit power to secondary users for utility enhancement; Secondary users, i.e., bidders, purchase power as well as channels from primary users for utility maximization. The prices announced by primary users for channels and cooperative power are determined by ascending clock auction algorithm. Also, we mathematically prove the convergence (i.e.,

the convergence to an unique Walrasian Equilibrium) of the proposed auction game, and show that the proposed auction is beneficial to both primary and secondary users by numerical results.

The contributions of our work are summarized as follows:

- 1) A new cooperative CR framework, i.e., primary users cooperate for secondary users, is proposed, where primary users earn the revenue by selling channels and cooperative power to secondary users. While secondary users maximize the utility by finding an optimal trade-off between the expense on channel and power and the achievable rate. Moreover, network coding is employed in cooperation for further diversity gain.
- 2) An auction-theoretic model with multiple auctioneers, multiple bidders and multiple commodities is developed for the proposed cooperative CR networks. Unlike traditional multi-commodity auction, the spectrum and cooperative power are not two independent commodities, but are offered as a bundle at each auctioneer. It requires each bidder to purchase the channel and power from the same auctioneer. Moreover, the spectrum and cooperative power are two different types of commodities. The channel is indivisible and thus can be assigned either totally or nothing, while the cooperative power is divisible and therefore can be offered at any quantity. All these properties of two commodities are specifically considered in our auction strategy. To the best of our knowledge, an auction model with multiple auctioneers, multiple bidders, and multiple heterogeneous bundling commodities for wireless resource allocation is reported for the first time in this paper.
- 3) The outcome of the proposed auction game is investigated, in which two important properties including the existence of an unique Walrasian Equilibrium and the efficiency of the convergence are theoretically proved.

The remainder of this paper is organized as follows: Section II presents the system model and the basic assumptions. Section III describes a joint spectrum and power auction mechanism for coded cooperative CR system. In Section IV, the convergence performance of the proposed auction game is theoretically analyzed. Numerical results are presented and discussed in Section V. Finally, Section VI concludes the paper.

II. NETWORK MODELING AND NOTATIONS

Consider a CR system consisting of a primary network composed of M primary links, and a secondary network composed of N secondary links. In the primary network, each primary user (PU) j is equipped with a primary transmitter (PT) and a primary receiver (PR), and has K_j non-overlapping narrowband channels. Assume that the channels owned by the same PU have the same carrier frequency, while those of different PUs are different so as to avoid interference of different primary transmissions. In the secondary network, each secondary user (SU) i is equipped with a secondary transmitter (ST) and a secondary receiver (SR). The PTs act as relays to assist SUs' transmissions by network coding and the amplify-and-forward relaying protocol. Also, assume that each channel of the PT

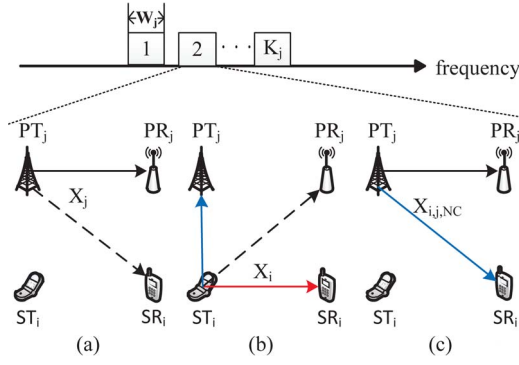


Fig. 1. Transmissions in three phases. (a) First phase. (b) Second phase. (c) Third phase.

can be accessed by only one ST at the same time, and the channel occupancy by the STs is maintained by each PT itself. For simplicity, we consider the scenario where the total number of channels in the system equals to the number of SUs, i.e., $N = \sum_{j=1}^M K_j$, such that each SU can access one channel.

The structure of our CR frame consists of an auction slot and a data transmission slot. In the auction slot, the ST, which intends to send data to its SR, selects a desired PT and joins the channel and power auction organized by that PT. The data transmission slot is further divided into three phases, as shown in Fig. 1. At each channel of PU j , in the first phase, PT j sends its data to its receiver. Meanwhile, the data are overheard by SR i who is designated to that channel by PT j ; In the second phase, ST i transmits its data to its receiver, which are also overheard by PR j ; In the third phase, PT j combines together its own data sent in the first phase and the data overheard in the second phase and sends the additive data out. Then both PR j and SR i can extract their desired data from the combined data by subtracting the data it overheard. Note that in Fig. 1, the solid lines indicate the intended communications, and the dotted lines represent the interference.

We now give out some operating assumptions of our system model: 1) Assume that the channels change slowly and the channel gain is stable within each frame. This assumption is widely used in the literature [22]–[24] for optimal resource allocation over wireless fading channels. It simplifies our analysis, but our work can be considered as a baseline analysis for more complicated scenarios. 2) Assume that the channel state information (CSI) can be accurately measured at each receiver, and this information can be sent to other receivers through an error-free control channel. Note that for slow-fading channel, i.e., the channel coherence time is large enough, the CSI can be accurately estimated within a sufficiently long period of observation.

In the First Phase: At each channel of PU j , PT j transmits its signal to its destination PR j with power P_j . Assume that the total transmit power P_{U_j} of PT j is equally used at each channel at this phase, i.e., we have $P_j = P_{U_j}/K_j$. The signals received at PR j and SR i are respectively given by

$$Y_{PT_j}^{PR_j} = \sqrt{P_j} G_{PT_j}^{PR_j} X_j + n_{PR_j}, \quad (1)$$

$$Y_{PT_j}^{SR_i} = \sqrt{P_j} G_{PT_j}^{SR_i} X_j + n_{SR_i}, \quad (2)$$

where Y_A^B represents the signal received at B from A , X_j is the information symbol transmitted by PT j with $E[|X_j|^2] = 1$, and $n_{\{\cdot\}}$ is the additive white Gaussian noise (AWGN) with variance σ^2 . G_A^B denotes the channel gains from A to B , which is also the channel gains from B to A . The amplitude $|G_A^B|^2$ is exponentially distributed, with rate parameter $\lambda_A^B = (d_A^B)^\alpha$, where d_A^B denotes the distance between A and B , and α is the path-loss exponent.

The signal-to-noise-ratio (SNR) of X_j at PR j in the first phase is

$$\Gamma_{PR_j}(1) = \frac{P_j |G_{PT_j}^{PR_j}|^2}{\sigma^2}. \quad (3)$$

In the Second Phase: ST i transmits its signal with power P_i . The signals received by SR i , PT j and PR j are

$$Y_{ST_i}^{SR_i} = \sqrt{P_i} G_{ST_i}^{SR_i} X_i + n_{SR_i}, \quad (4)$$

$$Y_{ST_i}^{PT_j} = \sqrt{P_i} G_{ST_i}^{PT_j} X_i + n_{PT_j}, \quad (5)$$

$$Y_{ST_i}^{PR_j} = \sqrt{P_i} G_{ST_i}^{PR_j} X_i + n_{PR_j}, \quad (6)$$

where X_i is the signal transmitted by ST i in this phase with $E[|X_i|^2] = 1$.

Thus, the SNR of X_i at SR i in the second phase is

$$\Gamma_{SR_i}(2) = \frac{P_i |G_{ST_i}^{SR_i}|^2}{\sigma^2}. \quad (7)$$

In the Third Phase: PT j makes a combination of $Y_{ST_i}^{PT_j}$ and its own signal X_j , and amplifies and forwards the combined signal $X_{i,j,NC}$. Then, the signals received by PR j and SR i are

$$Y_{PT_j,NC}^{PR_j} = \sqrt{P_{i,j}} G_{PT_j}^{PR_j} X_{i,j,NC} + n_{PR_j}, \quad (8)$$

$$Y_{PT_j,NC}^{SR_i} = \sqrt{P_{i,j}} G_{PT_j}^{SR_i} X_{i,j,NC} + n_{SR_i}, \quad (9)$$

where $P_{i,j}$ is PT j 's cooperative transmit power for ST i , and $X_{i,j,NC}$ is the normalized energy data symbol defined as

$$X_{i,j,NC} = \frac{X_j + \sqrt{P_i} G_{ST_i}^{PT_j} X_i + n_{PT_j}}{\sqrt{1 + P_i |G_{ST_i}^{PT_j}|^2 + \sigma^2}}. \quad (10)$$

The signal $Y_{PT_j,NC}^{PR_j}$ received at PR j contains the information of both X_j and X_i , where X_i is the interference signal overheard in the second phase that can be completely removed. This yields

$$\hat{Y}_{PT_j,NC}^{PR_j} = \frac{\sqrt{P_{i,j}} G_{PT_j}^{PR_j}}{\sqrt{1 + P_i |G_{ST_i}^{PT_j}|^2 + \sigma^2}} (X_j + n_{PT_j}) + n_{PR_j}. \quad (11)$$

In the third phase, the SNR at PR j is then given by

$$\Gamma_{PR_j}(3) = \frac{P_{i,j} |G_{PT_j}^{PR_j}|^2}{\sigma^2 \left(1 + P_i |G_{ST_i}^{PT_j}|^2 + P_{i,j} |G_{PT_j}^{PR_j}|^2 + \sigma^2 \right)}. \quad (12)$$

Correspondingly, the achievable rate from PT j to PR j when relaying for ST i is

$$R_{j,i} = \frac{W_j}{3} \log_2 (1 + \Gamma_{PR_j}(1) + \Gamma_{PR_j}(3)), \quad (13)$$

where W_j is the bandwidth of the channel. The factor $1/3$ comes from the fact that three phases are required to fulfill one cooperative transmission.

Therefore, the achievable rate from PT j to PR j , when relaying for K_j STs over all the channels, is

$$R_{j,C} = \sum_{i=1}^{K_j} R_{j,i} = \sum_{i=1}^{K_j} \frac{W_j}{3} \log_2 (1 + \Gamma_{PR_j}(1) + \Gamma_{PR_j}(3)). \quad (14)$$

Similarly, the useful signal, the SNR at SR i , and the achievable rate from ST i to SR i are respectively given by

$$\hat{Y}_{PT_j,NC}^{SR_i} = \frac{\sqrt{P_{i,j}} G_{PT_j}^{SR_i}}{\sqrt{1 + P_i |G_{ST_i}^{PT_j}|^2 + \sigma^2}} \times (\sqrt{P_i} G_{ST_i}^{PT_j} X_i + n_{PT_j}) + n_{SR_i}, \quad (15)$$

$$\Gamma_{SR_i}(3) = \frac{P_{i,j} |G_{PT_j}^{SR_i}|^2 P_i |G_{ST_i}^{PT_j}|^2}{\sigma^2 (1 + P_i |G_{ST_i}^{PT_j}|^2 + P_{i,j} |G_{PT_j}^{SR_i}|^2 + \sigma^2)}, \quad (16)$$

$$R_{i,j} = \frac{W_j}{3} \log_2 (1 + \Gamma_{SR_i}(2) + \Gamma_{SR_i}(3)). \quad (17)$$

In contrast, if PT j does not cooperate with any secondary user but occupy all the channels all the time itself (we refer to it as direct transmission in this paper), the achievable rate from PT j to PR j over all the channels becomes

$$R_{j,D} = K_j W_j \log_2 (1 + \Gamma_{PR_j}(1)). \quad (18)$$

III. JOINT SPECTRUM AND POWER AUCTION

In this section, we first introduce the utility functions for primary and secondary users to depict their satisfactions in the auction, and then propose a joint spectrum and power allocation scheme based on a multi-auctioneer, multi-bidder, and multi-commodity auction game.

A. PU's Utility Function

Each PU $j \in \{1, \dots, M\}$, sells two heterogeneous commodities (channels and cooperative power) among N SUs. The supply of PU j can be denoted by a vector $\mathbf{S}_j = (K_j, P_{U_j})$, which consists of the number of the licensed channels and the available cooperative power PU j has. The supply of the entire system is then denoted by $\mathbf{S} = (\mathbf{S}_1, \dots, \mathbf{S}_M)$. Let λ_j^1 and λ_j^2 be the prices of a channel and a power unit PU j asks for. The price

vector of PU j then is denoted by $\lambda_j = (\lambda_j^1, \lambda_j^2)$. It is noted that the channels owned by the same PU are assumed to be identical, i.e., they have the same bandwidth, carrier frequency, modulating scheme, etc., thus they sell at the same price.

The utility of PU j is defined as the summation of its achievable rate and the payoff it receives in channel and power auction. That is

$$U_{j,C}(\mathbf{S}_j, \lambda_j) = gR_{j,C} + \lambda_j^1 \mathbf{S}_j^T \\ = g \sum_{i=1}^{K_j} R_{j,i} + \lambda_j^1 K_j + \lambda_j^2 P_{U_j}. \quad (19)$$

where g is a positive constant providing conversion of units.

Similarly, we have the utility function of PU j in direct transmission as

$$U_{j,D} = gR_{j,D} = gK_j W_j \log_2 (1 + \Gamma_{PR_j}(1)). \quad (20)$$

B. SU's Utility Function

When purchasing the channel and cooperative power, each SU i wishes to maximize its transmission rate with minimum cost. Therefore, SU i requires to address two questions in the auction: 1) How to choose a PU from M PUs for cooperation? and 2) How much cooperative power should it request from that PU? Formally, we formulate the solutions of these two questions as the bids of SU i :

$$\mathbf{Q}_i = (\mathbf{Q}_{i,1}, \dots, \mathbf{Q}_{i,j}, \dots, \mathbf{Q}_{i,M})^T, \quad (21)$$

where $\mathbf{Q}_{i,j} = (C_{i,j}, P_{i,j})$, $\forall i \in \{1, \dots, N\}$, $\forall j \in \{1, \dots, M\}$, is a resource demand vector. $P_{i,j}$ represents the required cooperative power of SU i from PU j . $C_{i,j} \in \{0, 1\}$ specifies that whether SU i is willing to buy a channel from PU j . If it is, $C_{i,j} = 1$; Otherwise $C_{i,j} = 0$. It is worth mentioning that the channel and the cooperative power are two different types of commodities, of which the channel is indivisible and the cooperative power is divisible. Therefore, the channel is available in a supply of one and thus can be assigned either totally or nothing; The cooperative power can be offered at any quantity of $P_{i,j}$, subject to the constraint that $0 \leq \sum_{i=1}^N P_{i,j} \leq P_{U_j}$.

Furthermore, the proposed cooperation architecture requires the SU to buy the channel and the cooperative power from the same PU. It implies that the channel and the power cannot be sold independently at each PU, but should be offered as a bundle [27]. For each SU, it can only purchase the entire bundle or nothings. When SU i does not buy the channel from PU j , i.e., $C_{i,j} = 0$, it will not receive any power from that PU, i.e., we have $P_{i,j} = 0$. Similarly, if PU j does not assign any power to SU i , i.e., $P_{i,j} = 0$, it is not allowed to sell the channel to that SU, i.e., we have $C_{i,j} = 0$. Since each SU can access only one channel, we have $\sum_{j=1}^M C_{i,j} \leq 1$, $\forall i \in \{1, \dots, N\}$. Correspondingly, the power demand vector $\mathbf{P}_i = (P_{i,1}, P_{i,2}, \dots, P_{i,M})$ of each SU is a M -dimensional vector with at most one non-zero element.

To depict a SU's satisfaction with the received channel and power from PU j , we define a utility function of SU i to PU j as:

$$U_{i,j}(\mathbf{Q}_{i,j}, \boldsymbol{\lambda}_j) = gR_{i,j}(P_{i,j}) - \lambda_j^1 C_{i,j} - \lambda_j^2 P_{i,j}, \quad (22)$$

where in the right side of the equation, the first term is SU i 's gain (achievable rate) achieved in cooperation from PU j , and the second and the third term are the payment to PU j .

Then the utility function of SU i is defined as:

$$U_i = \max_{s \in \{1, \dots, M\}} U_{i,s}(\mathbf{Q}_{i,s}, \boldsymbol{\lambda}_s). \quad (23)$$

If SU i decides to purchase channel and power from PU j , i.e., we have $C_{i,j} = 1$, and $C_{i,k} = 0$ for any $k \neq j$, then the optimal cooperative power demand of SU i can be achieved by solving the following utility maximization problem:

$$\begin{aligned} \max_{P_{i,j}} \quad & U_{i,j}(\mathbf{Q}_{i,j}, \boldsymbol{\lambda}_j) = gR_{i,j}(P_{i,j}) - \lambda_j^1 - \lambda_j^2 P_{i,j} \\ \text{s.t.} \quad & 0 \leq P_{i,j} \leq P_{U_j}. \end{aligned} \quad (24)$$

It can be seen that the above maximization problem is a concave optimization problem as the objective function is strictly concave and the constraint set is convex [32]. We can find the optimal power demand $P_{i,j}^*$ by taking the derivative of $U_{i,j}(\mathbf{Q}_{i,j}, \boldsymbol{\lambda}_j)$ with respect to $P_{i,j}$ as

$$\frac{\partial U_{i,j}(\mathbf{Q}_{i,j}, \boldsymbol{\lambda}_j)}{\partial P_{i,j}} = g \frac{\partial R_{i,j}(P_{i,j})}{\partial P_{i,j}} - \lambda_j^2 = 0. \quad (25)$$

For simplicity, we define

$$\begin{aligned} W_j' &= \frac{gW_j}{3\ln(2)}; \quad \alpha_{ij} = 1 + \frac{P_i |G_{ST_i}^{SR_i}|^2}{\sigma^2} \\ \beta_{ij} &= \frac{P_i |G_{ST_i}^{PT_j}|^2}{\sigma^2}; \quad \gamma_{ij} = \frac{1 + P_i |G_{ST_i}^{PT_j}|^2 + \sigma^2}{|G_{PT_j}^{SR_i}|^2} \end{aligned} \quad (26)$$

Substituting (26) into (25), we have the optimal cooperative power demand $P_{i,j}^*(\lambda_j^2)$ shown in (27), shown at the bottom of the page.

C. Ascending Clock Auction Mechanism

We model a multi-auctioneer, multi-bidder and multi-commodity auction game to efficiently allocate the channels and cooperative power of M PUs among N SUs. Each PU j , i.e., the auctioneer, iteratively announces the prices of its two commodities. Each SU i , i.e., the bidder, responds to each PU j by submitting its demand $\mathbf{Q}_{i,j}$, which reports the quantities of the channel and the power it wishes to purchase at these

prices. PU j then calculates the *cumulative clinch* and credits the channel and the power to the SUs at the current prices by ascending clock auction algorithm [28]. Thereafter, PU j adjusts the prices according to the relationship between the total demand and the total supply. This process repeats until the prices converge at which the total demand is less than or equal to the total supply. During this process, several important operations including PU selection, reserve pricing, resource crediting, and payment calculation are involved.

1) *PU Selection*: The PU selection occurs on each SU at each auction clock, by which the SU determines from which PU it buys a channel and how much cooperative power it requests from that PU. For example, at the very beginning of the auction, i.e., at the time when the auction clock index τ is set to zero, each PU j makes an initialization and announces its initial prices in a form of $\boldsymbol{\lambda}_j(0) = (\lambda_j^1(0), \lambda_j^2(0))$ to all the SUs. Based on these prices, SU i selects the desired PU. To do that, SU i , sets $C_{i,j} = 1, \forall j \in \{1, \dots, M\}$, and separately solve M utility maximization problems defined in (24). Then it finds out the desired PU j that incurs the maximum utility, that is

$$j = \arg \max_{k \in \{1, \dots, M\}} U_{i,k}(\mathbf{Q}_{i,k}, \boldsymbol{\lambda}_k). \quad (28)$$

Thereafter, SU i places its bids to PU j as $C_{i,j}(\lambda_j^1(0)) = 1$, and $P_{i,j}(\lambda_j^2(0)) = P_{i,j}^*(\lambda_j^2(0))$ defined in (27). For any other PU k , $\forall k \in \{1, \dots, M\}$ and $k \neq j$, it sets the bids to $C_{i,k}(\lambda_k^1(0)) = 0$ and $P_{i,k}(\lambda_k^2(0)) = 0$.

2) *Reserve Pricing*: The reserve price refers to the lowest prices at which the PU is willing to sell the channels and power to the SUs. It guarantees a certain amount of the revenue for the PU, even when competition is weak. Generally, the reserve price can be defined as the expense of relaying for the SUs, which reflects the adverse effects of SUs' transmissions on PU's performance, such as device depreciation, channel occupation and power consumption [26].

In this work, the PU can choose either to use all the resource itself (i.e., direct transmission) or to sell a fraction of the resource to the SUs (i.e., cooperative transmission). To encourage the PU to share the resource with the SUs, its reserve price vector $\boldsymbol{\lambda}_{j,R}$ can be set in such a way that at which the utility of the PU achieved in cooperative transmission is no less than that in direct transmission. That is to say, the value of $\boldsymbol{\lambda}_{j,R}$ satisfies:

$$U_{j,C}(\mathbf{S}_j, \boldsymbol{\lambda}_{j,R}) \geq U_{j,D}. \quad (29)$$

The value of $U_{j,C}(\mathbf{S}_j, \boldsymbol{\lambda}_{j,R})$ varies when different set of K_j SUs access PU j 's channels. So, this value is uncertain before the auction, as the PU has no idea that which SUs would finally access its channels. However, the PU can estimate its lower bound $\overline{U_{j,C}}(\mathbf{S}_j, \boldsymbol{\lambda}_{j,R})$ by finding out the set \mathcal{D} of K_j SUs, with

$$P_{i,j}^*(\lambda_j^2) = \max \left[0, \min \left(\frac{\sqrt{\beta_{ij}^2 \gamma_{ij}^2 + \frac{4W_j'}{\lambda_j^2} (\alpha_{ij} \beta_{ij} \gamma_{ij} + \beta_{ij}^2 \gamma_{ij})} - (2\alpha_{ij} \gamma_{ij} + \beta_{ij} \gamma_{ij})}{2(\alpha_{ij} + \beta_{ij})}, P_{U_j} \right) \right] \quad (27)$$

which PU j achieves the minimum transmission rates $\overline{R_{j,C}}$. That is

$$\overline{U_{j,C}}(\mathbf{S}_j, \boldsymbol{\lambda}_{j,R}) \approx g\overline{R_{j,C}} + \lambda_{j,R}^1 K_j + \lambda_{j,R}^2 P_{U_j}, \quad (30)$$

where we have $\overline{R_{j,C}} = \sum_{i \in \mathcal{D}} R_{j,i}$, in which $R_{j,i}$ is calculated before the auction under the assumption that PU j equally allocates its cooperative power among all the channels, i.e., $P_{i,j} = P_{U_j}/K_j$.

Then we set

$$\overline{U_{j,C}}(\mathbf{S}_j, \boldsymbol{\lambda}_{j,R}) = U_{j,D}. \quad (31)$$

Combining (14) and (20) into (31), we have

$$\begin{aligned} \sum_{i \in \mathcal{D}} \frac{gW_j}{3} \log_2 (1 + \Gamma_{PR_j}(1) + \Gamma_{PR_j}(3)) + \lambda_{j,R}^1 K_j \\ + \lambda_{j,R}^2 P_{U_j} = \frac{gK_j W_j}{3} \log_2 (1 + 3\Gamma_{PR_j}(1)) \end{aligned} \quad (32)$$

We cannot acquire $\lambda_{j,R}^1$ and $\lambda_{j,R}^2$ simultaneously from (32). One simplest solution is to set $\lambda_{j,R}^1 = 0$ and get $\lambda_{j,R}^2$ by

$$\begin{aligned} \lambda_{j,R}^2 = \frac{1}{P_{U_j}} \left[\frac{gK_j W_j}{3} \log_2 (1 + 3\Gamma_{PR_j}(1)) \right. \\ \left. - \sum_{i \in \mathcal{D}} \frac{gW_j}{3} \log_2 (1 + \Gamma_{PR_j}(1) + \Gamma_{PR_j}(3)) \right]. \end{aligned} \quad (33)$$

The reserve price is very easily implemented in ascending clock auction. For example, the PU can initialize $\boldsymbol{\lambda}_j(0)$ to $\boldsymbol{\lambda}_{j,R}$, such that it can always get a larger utility from choosing cooperation.

3) *Resource Crediting*: At each auction clock $\tau = 0, 1, \dots$, PU j collects N SUs' bids, and computes the total required channels and power of these SUs. Let $C_j^{\text{tal}}(\lambda_j^1(\tau)) = \sum_{i=1}^N C_{i,j}(\lambda_j^1(\tau))$ and $P_j^{\text{tal}}(\lambda_j^2(\tau)) = \sum_{i=1}^N P_{i,j}(\lambda_j^2(\tau))$ represent the total channel and power demand at PU j at clock τ , respectively. Further, let $E_j^1(\lambda_j^1(\tau)) = C_j^{\text{tal}}(\lambda_j^1(\tau)) - K_j$ and $E_j^2(\lambda_j^2(\tau)) = P_j^{\text{tal}}(\lambda_j^2(\tau)) - P_{U_j}$ represent the excess channel and power demand at PU j , respectively. Then PU j adjusts its price vector according to the excess demand.

Case 1: $E_j^1(\lambda_j^1(\tau)) > 0$ and $E_j^2(\lambda_j^2(\tau)) > 0$: It tells that the total demand for the power as well as for the channel exceeds the supply. Due to the indivisibility, K_j channels cannot be divided and fairly allocated among more than K_j competitors. Therefore, none of the channels would be credited to any competitor. For bundling sale, the power would not be credited to any competitor, either. So we have

$$\hat{C}_{i,j}(\lambda_j^1(\tau)) = 0, \quad \hat{P}_{i,j}(\lambda_j^2(\tau)) = 0, \quad \forall i \in \{1, 2, \dots, N\}, \quad (34)$$

where $\hat{C}_{i,j}(\lambda_j^1(\tau))$ and $\hat{P}_{i,j}(\lambda_j^2(\tau))$ are the cumulative clinch, which are the amounts of the channel and power that are credited to SU i at the price $\boldsymbol{\lambda}_j(\tau)$.

After finishing the computation of the cumulative clinch, PU j updates its price vector with $\lambda_j^1(\tau + 1) = \lambda_j^1(\tau) + \mu_j^1$, and $\lambda_j^2(\tau + 1) = \lambda_j^2(\tau) + \mu_j^2$, where $\mu_j^1 > 0$ and $\mu_j^2 > 0$ are step

sizes, and announces this new price vector to all SUs. Each SU then re-selects a PU based on the new announced prices and starts a new bidding round.

Case 2: $E_j^1(\lambda_j^1(\tau)) > 0$ and $E_j^2(\lambda_j^2(\tau)) \leq 0$: In this case, there are more than K_j SUs competing for K_j channels at PU j , whose total power demand is less than PU j 's supply. Similar to Case 1, neither the channel nor the power would be credited to any competitor. Therefore, the cumulative clinch of the channel and power to the SUs are also calculated by (34). Finally, the price of the channel is updated by $\lambda_j^1(\tau + 1) = \lambda_j^1(\tau) + \mu_j^1$. While the price of per unit power remains unchanged as $\lambda_j^2(\tau + 1) = \lambda_j^2(\tau)$ for the sake that the total power demand does not exceed the supply.

Case 3: $E_j^1(\lambda_j^1(\tau)) \leq 0$ and $E_j^2(\lambda_j^2(\tau)) > 0$: In this case, the competition for the power is fierce, while that for the channel is weak. As the supply of the channels is sufficient, the channels can be credited to all the competitors who bid for them. Moreover, the power can be credited to each competitor in terms of their opponents' demands. Thus, for each SU i with $C_{i,j}(\lambda_j^1(\tau)) = 1$, we have

$$\begin{aligned} \hat{C}_{i,j}(\lambda_j^1(\tau)) = 1, \\ \hat{P}_{i,j}(\lambda_j^2(\tau)) = \max \left(0, P_{U_j} - \sum_{k=1, k \neq i}^N P_{k,j}(\lambda_j^2(\tau)) \right). \end{aligned} \quad (35)$$

For each SU i with $C_{i,j}(\lambda_j^1(\tau)) = 0$, we have

$$\hat{C}_{i,j}(\lambda_j^1(\tau)) = 0, \quad \hat{P}_{i,j}(\lambda_j^2(\tau)) = 0. \quad (36)$$

Thereafter, PU j updates the price vector with $\lambda_j^1(\tau + 1) = \lambda_j^1(\tau)$, and $\lambda_j^2(\tau + 1) = \lambda_j^2(\tau) + \mu_j^2$.

Case 4: $E_j^1(\lambda_j^1(\tau)) \leq 0$ and $E_j^2(\lambda_j^2(\tau)) \leq 0$: It shows that the supply of both channel and power is sufficient for all the competitors. Therefore, each competitor would be credited according to its demand. Namely

$$\hat{C}_{i,j}(\lambda_j^1(\tau)) = C_{i,j}(\lambda_j^1(\tau)), \quad \hat{P}_{i,j}(\lambda_j^2(\tau)) = P_{i,j}(\lambda_j^2(\tau)). \quad (37)$$

Then two prices are kept unchanged with $\lambda_j^1(\tau + 1) = \lambda_j^1(\tau)$, and $\lambda_j^2(\tau + 1) = \lambda_j^2(\tau)$.

Additionally, the demand $\mathbf{Q}_{i,j}(\boldsymbol{\lambda}_j(\tau))$ of SU i from PU j is a function of PU j 's announced price $\boldsymbol{\lambda}_j(\tau)$. If PU j 's price is too high at τ , SU i which chose PU j at $\tau - 1$ might give up it and choose another PU at τ , then all the channel and power clinched to SU i before at PU j become unclinched. Thus, all the credits SU i received before from PU j should be cleared. So, in the above four cases, for SU i with $C_{i,j}(\lambda_j^1(\tau)) = 0$ and $C_{i,j}(\lambda_j^1(\tau - 1)) = 1$, we have

$$\hat{C}_{i,j}(\lambda_j^1(\tau')) = 0, \quad \hat{P}_{i,j}(\lambda_j^2(\tau')) = 0, \quad \forall \tau' \in \{0, 1, \dots, \tau - 1\}. \quad (38)$$

4) *Payment Calculation*: Assuming that the supply meets the total demand for each PU at clock $\tau = T$, i.e., $E_j^1(\lambda_j^1(T)) = 0$ and $E_j^2(\lambda_j^2(T)) \leq 0$, $\forall j \in \{1, \dots, M\}$, then the auction converges to an equilibrium price vector $\boldsymbol{\lambda}_j^* = \boldsymbol{\lambda}_j(T)$. Consider that the supply P_{U_j} might not be fully covered at

λ_j^* , i.e., $E_j^2(\lambda_j^2(\tau)) < 0$. For each SU i with $\hat{P}_{i,j}(\lambda_j^2(T)) \neq 0$, its cumulative clinch is re-calculated by [28]:

$$\begin{aligned} \hat{P}_{i,j}(\lambda_j^2(T)) &= P_{i,j}(\lambda_j^2(T)) \\ &+ \frac{P_{i,j}(\lambda_j^2(T-1)) - P_{i,j}(\lambda_j^2(T))}{\sum_{i=1}^N P_{i,j}(\lambda_j^2(T-1)) - \sum_{i=1}^N P_{i,j}(\lambda_j^2(T))} \\ &\times \left[P_{U_j} - \sum_{i=1}^N P_{i,j}(\lambda_j^2(T)) \right]. \end{aligned} \quad (39)$$

Such that we have $E_j^1(\lambda_j^1(T)) = 0$ and $E_j^2(\lambda_j^2(T)) = 0$.

Finally, the quantities of the channel and power that are assigned to the SU are given by

$$\begin{aligned} C_{i,j}^* &= \hat{C}_{i,j}(\lambda_j^1(T)), \quad P_{i,j}^* = \hat{P}_{i,j}(\lambda_j^2(T)), \\ \forall i &\in \{0, 1, \dots, N\}, \quad \forall j \in \{0, 1, \dots, M\}. \end{aligned} \quad (40)$$

Correspondingly, the payment for the channel from SU i to PU j is

$$\begin{aligned} V_{i,j}^1 &= \lambda_j^1(0) \hat{C}_{i,j}(\lambda_j^1(0)) \\ &+ \sum_{\tau=1}^T \lambda_j^1(\tau) \left(\hat{C}_{i,j}(\lambda_j^1(\tau)) - \hat{C}_{i,j}(\lambda_j^1(\tau-1)) \right), \end{aligned} \quad (41)$$

and the payment for the power from SU i to PU j is

$$\begin{aligned} V_{i,j}^2 &= \lambda_j^2(0) \hat{P}_{i,j}(\lambda_j^2(0)) \\ &+ \sum_{\tau=1}^T \lambda_j^2(\tau) \left(\hat{P}_{i,j}(\lambda_j^2(\tau)) - \hat{P}_{i,j}(\lambda_j^2(\tau-1)) \right). \end{aligned} \quad (42)$$

A complete channel and power auction algorithm is shown in Algorithm 1. The communication overhead arise from the transmissions of the bids from the SU and the price vector from the PU, which are negligible compared to the main traffic. Since bids from different SUs may arrive at the PU at different time, the proposed auction algorithm can also run in an asynchronous way. At each auction clock, the PU collects new bids until a timeout value \bar{T} has passed. For the SUs whose bids it has received, it uses the new bids; For those slow ones that it has not heard from in this round, it uses the most recent bids from them.

Algorithm 1 The Proposed Channel and Power Auction Algorithm

Initialization

Sets clock index $\tau = 0$;

Each PU $j \in \{1, \dots, M\}$ announces its initial price vector $\lambda_j(0) = (\lambda_j^1(0), \lambda_j^2(0))$ to all the SUs.

Iteration Step ($\tau = 0, 1, 2, \dots$)

At a SU $i \in \{1, \dots, N\}$:

Input: Receives the price vector $\lambda_j(\tau) = (\lambda_j^1(\tau), \lambda_j^2(\tau))$ from each PU $j \in \{1, \dots, M\}$.

Finds out the desired PU j according to (28);

Calculates the optimal power demand $P_{i,j}^*(\lambda_j^2(\tau))$ in (27) from PU j ;

Sets the bid $\mathbf{Q}_{i,j}(\tau) = (1, P_{i,j}^*(\lambda_j^2(\tau)))$ to PU j ;

For any other PU $k \neq j$, sets the bid $\mathbf{Q}_{i,k}(\tau) = (0, 0)$.

Output: The new bid $\mathbf{Q}_{i,j}(\tau) = (C_{i,j}(\lambda_j^1(\tau)), P_{i,j}(\lambda_j^2(\tau)))$ to each PU.

At a PU $j \in \{1, \dots, M\}$:

Input: Collects the bid $\mathbf{Q}_{i,j}(\tau) = (C_{i,j}(\lambda_j^1(\tau)), P_{i,j}(\lambda_j^2(\tau)))$ from each player $i \in \{1, \dots, N\}$.

Calculates the excess channel demand by $E_j^1(\lambda_j^1(0)) =$

$\sum_{i=1}^N C_{i,j}(\lambda_j^1(0)) - K_j$;

Calculates the excess power demand by $E_j^2(\lambda_j^2(0)) =$

$\sum_{i=1}^N P_{i,j}(\lambda_j^2(0)) - P_{U_j}$;

If $E_j^1(\lambda_j^1(\tau)) > 0$ and $E_j^2(\lambda_j^2(\tau)) > 0$

$\hat{C}_{i,j}(\lambda_j^1(\tau)) = 0, \hat{P}_{i,j}(\lambda_j^2(\tau)) = 0, \forall i \in \{1, 2, \dots, N\}$;

$\lambda_j^1(\tau+1) = \lambda_j^1(\tau) + \mu_j^1, \lambda_j^2(\tau+1) = \lambda_j^2(\tau) + \mu_j^2$;

Else if $E_j^1(\lambda_j^1(\tau)) > 0$ and $E_j^2(\lambda_j^2(\tau)) \leq 0$

$\hat{C}_{i,j}(\lambda_j^1(\tau)) = 0, \hat{P}_{i,j}(\lambda_j^2(\tau)) = 0, \forall i \in \{1, 2, \dots, N\}$;

$\lambda_j^1(\tau+1) = \lambda_j^1(\tau) + \mu_j^1, \lambda_j^2(\tau+1) = \lambda_j^2(\tau)$;

Else if $E_j^1(\lambda_j^1(\tau)) \leq 0$ and $E_j^2(\lambda_j^2(\tau)) > 0$

$\hat{C}_{i,j}(\lambda_j^1(\tau)) = 0, \hat{P}_{i,j}(\lambda_j^2(\tau)) = 0$, for SU i with

$C_{i,j}(\lambda_j^1(\tau)) = 1$;

$\lambda_j^1(\tau+1) = \lambda_j^1(\tau), \lambda_j^2(\tau+1) = \lambda_j^2(\tau) + \mu_j^2$;

Else

$\hat{C}_{i,j}(\lambda_j^1(\tau)) = C_{i,j}(\lambda_j^1(\tau)), \hat{P}_{i,j}(\lambda_j^2(\tau)) = P_{i,j}(\lambda_j^2(\tau))$,

$\forall i \in \{1, 2, \dots, N\}$;

For SU i with $C_{i,j}(\lambda_j^1(\tau)) = 0$ and $C_{i,j}(\lambda_j^1(\tau-1)) = 1$,

sets $\hat{C}_{i,j}(\lambda_j^1(\tau')) = 0, \hat{P}_{i,j}(\lambda_j^2(\tau')) = 0, \forall \tau' \in$

$\{0, 1, \dots, \tau-1\}$;

Output: The new price vector $\lambda_j(\tau+1) = (\lambda_j^1(\tau+1), \lambda_j^2(\tau+1))$ to all SUs.

Iterates until the price vectors of all the PUs converge at $\tau = T$, then proceeds to the final step.

Final Step

For SU i with $\hat{P}_{i,j}(\lambda_j^2(T)) \neq 0$, each PU j updates the cumulative clinch $\hat{P}_{i,j}(\lambda_j^2(T))$ by (39);

Calculates the quantities of the channel and power that are assigned to each SU i by (40).

The computational complexity at the SU is mainly determined by the calculation of $P_{i,j}^*$ in (27). At each iteration, SU i requires 21 multiplications and 5 summations [33] to find out the $P_{i,j}^*$ for PU j . Assuming that $P_{i,j}^*$ converges after $I_{i,j}$ iterations (The convergence of the proposed auction algorithm is showed in Section IV), then the total multiplications and summations needed for the proposed algorithm are $J_M = \sum_{i=1}^N \sum_{j=1}^M 21I_{i,j}$ and $J_S = \sum_{i=1}^N \sum_{j=1}^M 5I_{i,j}$. Let $I^* = \max_{1 \leq i \leq N, 1 \leq j \leq M} I_{i,j}$, then we have $J_M \leq 21I^*MN$ and $J_S \leq 5I^*MN$. Therefore, the computational complexity of the proposed algorithm is $O(MN)$.

IV. THEORETIC ANALYSIS

First, we specify a generic economic model: M auctioneers wish to allocate K types of commodities among N bidders. For each auctioneer j , its available supply is $\mathbf{S}_j = (S_j^1, \dots, S_j^K)$, its announced price vector is $\lambda_j = (\lambda_j^1, \dots, \lambda_j^K)$, and its allocation to bidder i is $\mathbf{A}_{i,j} = (A_{i,j}^1, \dots, A_{i,j}^K)$. For each bidder i , its demand from auctioneer j at price λ_j is $\mathbf{Q}_{i,j}(\lambda_j) = (Q_{i,j}^1(\lambda_j^1), \dots, Q_{i,j}^K(\lambda_j^K))$, its payment to auctioneer j is $V_{i,j}$, and it has a function $F_{i,j}(\mathbf{Q}_{i,j})$ with respect to $\mathbf{Q}_{i,j}$.

We now discuss two important properties of the proposed auction algorithm.

A. Existence of Walrasian Equilibrium

Definition 1. [29]: A Walrasian Equilibrium is a $M \times K$ price vector $\lambda^* = \begin{pmatrix} \lambda_1^{1*} & \lambda_1^{2*} & \dots & \lambda_1^{K*} \\ \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_M^{1*} & \lambda_M^{2*} & \dots & \lambda_M^{K*} \end{pmatrix}$ and a $N \times M \times K$ allocation vector $\mathbf{A}^* = \begin{pmatrix} \mathbf{A}_{1,1}^* & \mathbf{A}_{1,2}^* & \dots & \mathbf{A}_{1,M}^* \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{A}_{N,1}^* & \mathbf{A}_{N,2}^* & \dots & \mathbf{A}_{N,M}^* \\ \mathbf{Q}_{1,1}(\lambda^*) & \mathbf{Q}_{1,2}(\lambda^*) & \dots & \mathbf{Q}_{1,M}(\lambda^*) \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{Q}_{N,1}(\lambda^*) & \mathbf{Q}_{N,2}(\lambda^*) & \dots & \mathbf{Q}_{N,M}(\lambda^*) \end{pmatrix}$, such that $\mathbf{Q}(\lambda^*) = \mathbf{A}^*$, and $S_j^k = \sum_{i=1}^N A_{i,j}^{k*}, \forall j \in \{1, \dots, M\}, \forall k \in \{1, \dots, K\}$.

According to Definition 1, when the auction reaches a Walrasian Equilibrium, the excess demand of each bidder is zero, and the aggregate demand equals to the supply for each commodity.

Lemma 1. [28]: An auction has a Walrasian Equilibrium if it satisfies:

- 1) *Pure private values:* Bidder i 's value, i.e., $F_{i,j}(\mathbf{Q}_{i,j})$, for the demand vector $\mathbf{Q}_{i,j}$ does not change when bidder i learns other bidders' information.
- 2) *Quasilinearity:* Bidder i 's utility from receiving the demand vector $\mathbf{Q}_{i,j}$ in return for the payment $V_{i,j}$ is given by $F_{i,j}(\mathbf{Q}_{i,j}) - V_{i,j}$.
- 3) *Monotonicity:* The function $F_{i,j}(\mathbf{Q}_{i,j})$ is increasing, i.e., if $\mathbf{Q}'_{i,j} > \mathbf{Q}_{i,j}$, then $F_{i,j}(\mathbf{Q}'_{i,j}) > F_{i,j}(\mathbf{Q}_{i,j})$.
- 4) *Concavity:* The function $F_{i,j}(\mathbf{Q}_{i,j})$ is concave.

Theorem 1: The proposed multi-auctioneer multi-bidder and multi-commodity auction has a Walrasian Equilibrium.

Proof: Pure private values: In the proposed auction, bidder i 's demand $\mathbf{Q}_{i,j}$ is a function of the price vector λ_j . That is to say, the value of $R_{i,j}(\mathbf{Q}_{i,j})$ is uniquely determined by the announced price vector λ_j . As long as λ_j is fixed, \mathbf{Q}_i would always remain unchanged, regardless of the demands of other bidders.

Quasilinearity: It is obvious that the utility function $U_{i,j}(\mathbf{Q}_{i,j}, \lambda_j) = gR_{i,j}(P_{i,j}) - \lambda_j^1 C_{i,j} - \lambda_j^2 P_{i,j}$ in (21) is a linear function of the price λ_j^1 and λ_j^2 .

Monotonicity: It can be easily found that $\partial R_{i,j}(P_{i,j}) / \partial P_{i,j} > 0$, therefore, the achievable rate function $R_{i,j}(P_{i,j})$ is increasing.

Concavity: Note that if a function $f(x)$ is twice-differentiable, then $f(x)$ is strictly concave if and only if $f''(x)$ is negative. Since we have $\partial^2 R_{i,j}(P_{i,j}) / \partial^2 P_{i,j} < 0$, the achievable rate function $R_{i,j}(P_{i,j})$ is concave with respect to $P_{i,j}$.

Therefore, there exists a Walrasian Equilibrium for the proposed auction. ■

Corollary 1: Assume that the proposed auction has an Equilibrium price vector λ^* and an Equilibrium allocation vector \mathbf{A}^* . For any allocation vector $\mathbf{A} \neq \mathbf{A}^*$, $U_i(\lambda^*, \mathbf{A}^*) \geq U_i(\lambda^*, \mathbf{A})$ holds for each bidder $i \in \{1, \dots, N\}$.

B. Convergence

Theorem 1 shows the existence of a Walrasian Equilibrium for the proposed auction algorithm, but it does not tell us how it can efficiently converge to an equilibrium. As previous mentioned, the price adjustment of the proposed auction is directly controlled by the excess demand. If there is excess demand (e.g., for the power) at PU j , i.e., $E_j^2(\lambda_j^2(\tau)) > 0$, the price is increased by μ_j^2 . Otherwise, it keeps fixed. In fact, such price adjustment mechanism can be viewed as a discrete version of the Walrasian tâtonnement [30], i.e.,

$$\lambda_j^k(\tau + 1) = \lambda_j^k(\tau) + \lambda_j^k(\tau), \quad (43)$$

where we have

$$\lambda_j^k(\tau) = \begin{cases} \mu_j^k, & \text{if } E_j^k(\lambda_j^k(\tau)) > 0; \\ 0, & \text{otherwise.} \end{cases} \quad (44)$$

This ascending clock process continually drive the price vector λ to converge to λ^* , at which $\mathbf{E}(\lambda^*) = \mathbf{0}$.

Mathematically, we use the following Lyapunov stability theorem [31] to prove the convergence of the proposed algorithm.

Lemma 2 (Lyapunov's Theorem): Consider an autonomous system and its equilibrium point $\dot{x} = 0$. This equilibrium point is globally stable if there exists a Lyapunov function $V(x)$, which is continuously differentiable, such that

- 1) $V(x) > 0, \forall x \neq 0$; (positive definite)
- 2) $\dot{V}(x) \leq 0, \forall x$; (seminegative definite)
- 3) $V(x) \rightarrow \infty$, when $\|x\| \rightarrow \infty$.

Theorem 2: Starting from any sufficiently small price vector $\lambda(\mathbf{0})$, the proposed auction algorithm converges to a Walrasian Equilibrium price vector λ^* in finite iterations.

Proof: According to Lyapunov's Theorem, if we can find a Lyapunov function for the proposed algorithm (which can be considered as a nonlinear autonomous system), such that all three conditions are satisfied, then the equilibrium point of the

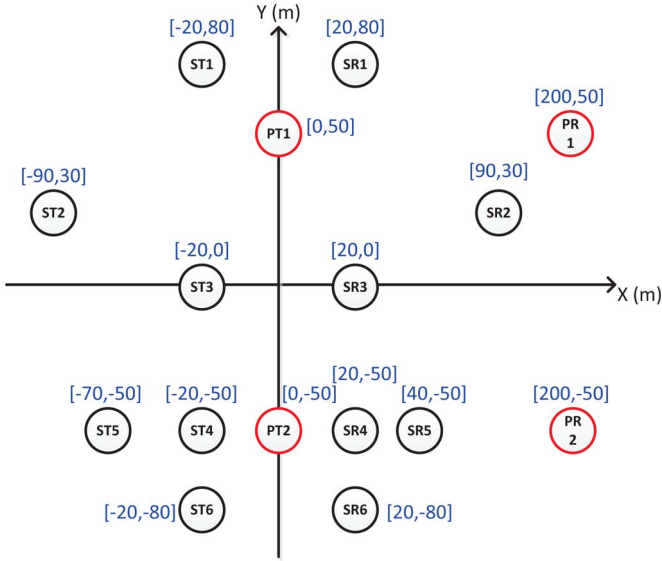


Fig. 2. A six-SU two-PU simulation network.

dynamical system is globally asymptotically stable. Similar to [28], we define the Lyapunov function as:

$$L(\lambda(\tau)) = \lambda(\tau) \cdot \mathbf{S} + \sum_{i=1}^N U_i(\mathbf{Q}_i, \lambda(\tau)). \quad (45)$$

It's easy to see that $L(\lambda) > 0$ when $\lambda \neq 0$, and $L(\lambda) \rightarrow \infty$, when $\|\lambda\| \rightarrow \infty$. By taking the derivative of this Lyapunov function, we have

$$\begin{aligned} \dot{L}(\lambda(\tau)) &= \frac{\partial L(\lambda(\tau))}{\partial \lambda(\tau)} = \mathbf{S} \cdot \dot{\lambda}(\tau) - \sum_{i=1}^N \mathbf{Q}_i(\lambda(\tau)) \cdot \dot{\lambda}(\tau) \\ &= \left(\mathbf{S} - \sum_{i=1}^N \mathbf{Q}_i(\lambda(\tau)) \right) \cdot \dot{\lambda}(\tau) \\ &= -\mathbf{E}(\lambda(\tau)) \cdot \dot{\lambda}(\tau). \end{aligned} \quad (46)$$

Clearly, when the demand exceeds the supply, i.e., $\mathbf{E}(\lambda(\tau)) > \mathbf{0}$, the price increases at τ , i.e., $\dot{\lambda}(\tau) > \mathbf{0}$, we then have $\dot{L}(\lambda(\tau)) < 0$. If the supply meets the demand and thus $\lambda(\tau) \rightarrow \lambda^*$, we have $\dot{\lambda}(\tau) = \mathbf{0}$, and $\dot{L}(\lambda(\tau)) = 0$. Therefore, the semi-negative definite condition $\dot{L}(\lambda(\tau)) \leq 0$ is satisfied. ■

V. SIMULATION RESULTS

In this section, we present simulation results to demonstrate the performance of the proposed joint spectrum and power allocation algorithm. We consider a scenario as shown in Fig. 2, where there are two PUs and six SUs in the network. PU 1 has 2 channels and PU 2 has 4 channels. The channel gains are $(0.097/d^\alpha)^{1/2}$, where d is the distance between two nodes, and the path-loss exponent is $\alpha = 4$. The transmit power of each SU is 0.01 W, and the noise variance is $\sigma^2 = 10^{-13}$. Without a special specification, the transmit power of PU 1 is 2 W, the transmit power of PU 2 is 1 W, the initial power and channel price of PU 1 are 0.8, and the initial power and channel price of PU 2 are 1.

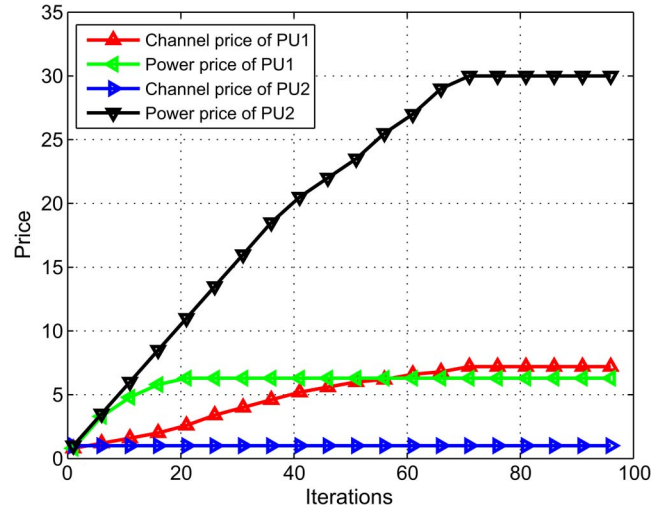


Fig. 3. Convergence of PUs' prices.

Fig. 3 shows the convergence performance of the proposed allocation algorithm, where the step sizes of both PUs' channel prices are set to $\mu_1^1 = \mu_2^1 = 0.2$, and the step sizes of both PUs' power prices are set to $\mu_1^2 = \mu_2^2 = 0.5$. It is observed that four prices converge at different speed, and the entire auction game converges after 71 iterations. Compared the convergent power prices (i.e., the optimal values) of two PUs, we find that the optimal power price of PU 2 is higher than that of PU 1. It indicates that the power competition at PU 2 is stronger than that at PU 1, thus leading to more auction clocks and a higher convergent power price. Also, it is noticed that the channel price of PU 2 remains unchanged throughout the entire auction process. The reason is that the number of SUs that choose PU 2 is always no more than the number of PU 2's available channels. Therefore, the channel price of PU 2 keeps at the initial price during the whole auction.

To evaluate the impact of the step size on the convergence performance, we set the step sizes of two PUs' power prices to 0.005 and 1, respectively. As can be seen in Fig. 4, the power prices converge much fast with larger step sizes than with smaller step sizes. For example, the power price of PU 1 takes only 15 iterations to reach the convergence when its step size is 1. In contrast, it needs 2329 iterations to converge when the step size is 0.005. However, a smaller step size drives the auction to converge to the same point achieved by the centralized optimization algorithm [32], while a larger step size can only approximates to this optimum.

Fig. 5 displays the evolution of the PU selection at SU 3, where "1" (Y-axis) represents SU 3 selects PU 1, and "2" represents it chooses PU 2. It is found that SU 3's selection varies between PU 1 and PU 2, and is finally stable at PU 2. According to the proposed algorithm, the SU at each auction clock, chooses the PU that currently incur the maximum utility. Due to different excess demand, two PUs might conduct different price adjustments at each clock. As a result, the PU selected by SU 3 at clock τ might not bring it the maximum utility at $\tau + 1$, then SU 3 leaves that PU and selects another one at $\tau + 1$. For instance, SU 3 selects PU 2 at $\tau = 9$ and changes to PU 1 at $\tau = 10$.

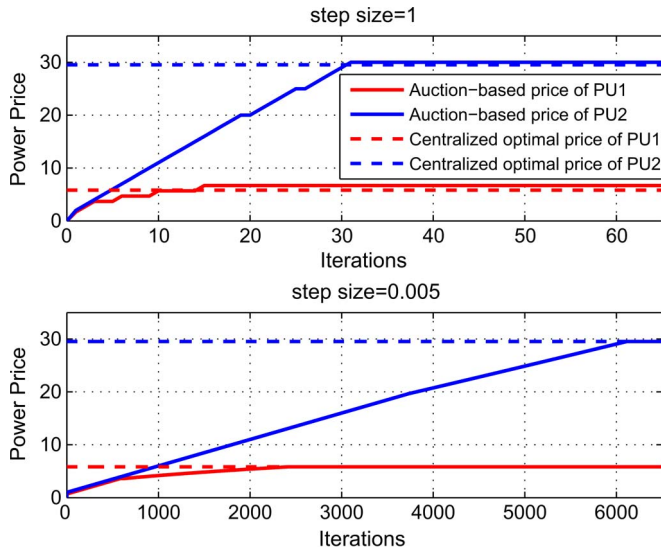


Fig. 4. Impact of the step size on the convergence performance.

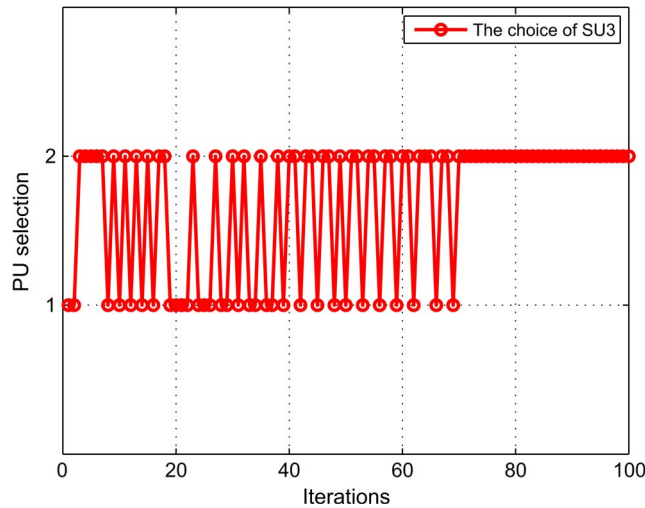


Fig. 5. The PU selection at SU 3.

We now adjust the transmit power supplies of two PUs simultaneously within a range of [1 W, 7 W], and keep other settings unchanged. It is seen in Fig. 6, that the optimal power price of each PU decreases with the increase of its power supply. This is due to the fact that the less power available at a PU, the smaller possibility that this supply can meet the total power demand of the SUs, the higher optimal power price would be, and vice versa.

Figs. 7 and 8 display the cooperative power and the corresponding utility achieved at each SU, where the transmit power supplies of two PUs are simultaneously increased from 1 W to 7 W. Note that in these cases, SU 1 and SU 2 always choose PU 1, and SU 3~6 always choose PU 2. It is observed that the cooperative power allocated to each SU increases with the power supply of each PU. On one hand, the more power available at the PU, the more power is assigned to each SU, and the higher achievable rate each SU achieves. On the other hand, the power price decreases as the power supply increases. As a

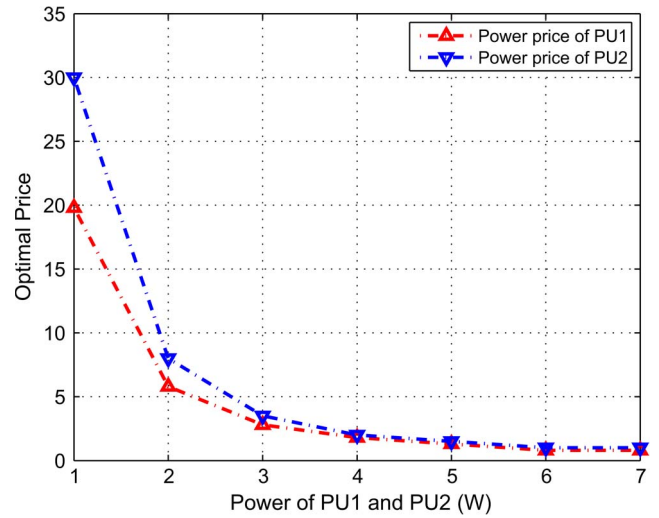


Fig. 6. Optimal power price of each PU.

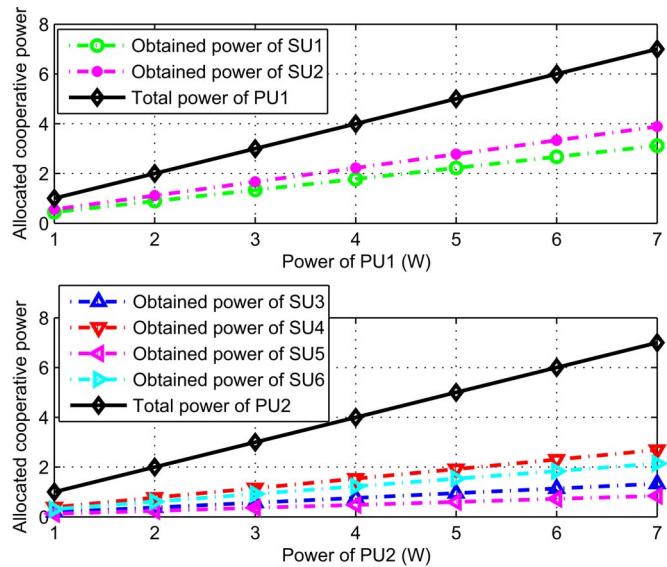


Fig. 7. The cooperative power allocated to SUs.

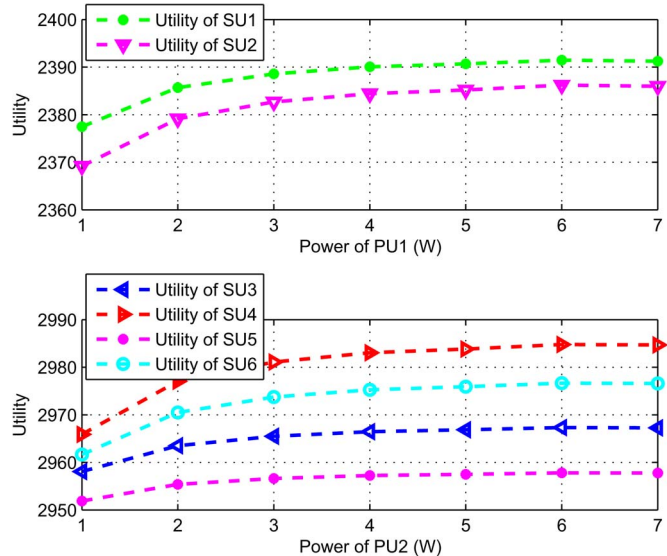


Fig. 8. The utility achieved at SUs.

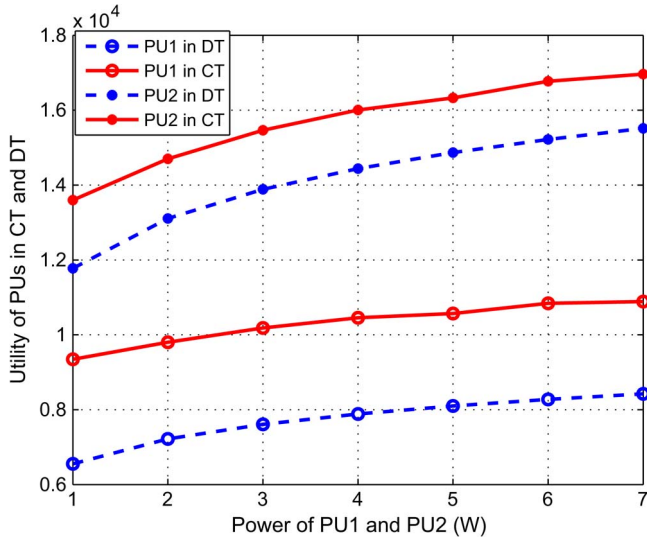


Fig. 9. Cooperative transmission vs. direct transmission for the PU.

result, the utility of each SU increases with the power supply of the PU.

Fig. 9 compares the utilities of PUs achieved in cooperative transmission (CT) with those in direct transmission (DT), where the initial channel prices of two PUs are set to 0, and their initial power prices are determined by (33). Such initial prices ensure that the utility of the PU in cooperative transmission is no less than that in direct transmission from the very beginning of the auction. With the ascending clock, the utilities of two PUs in cooperative transmission are always larger than in direct transmission.

Fig. 10 shows the impacts of the initial power price on PU's utility achieved in CT, where the power supplies of two PUs are simultaneously set to 1 W, 4 W, and 7 W, respectively, and the initial channel prices of two PUs are set to 0. It is observed that each PU's utility in both CT and DT increases with the power supply. When we fix the power supply and adjust PU's initial power price, we find that PU's utility in CT is smaller than that in DT when the initial price is less than the reserved price. As the initial price becomes higher than the reserved price, the utility in CT is larger than that in DT. Once the increasing initial price exceeds a point at which the SUs quit the auction for a non-positive utility, the PU's utility in CT comes to zero.

Fig. 11 compares the utilities of SUs achieved in cooperative transmission (CT) with those in direct transmission (DT), where the power supply of each PU is 5 W. Note that SU in CT refers to that 6 SUs work in the proposed cooperative way, and SU in DT refers to that these SUs only uses 6 channels of two PUs and sends the data to the receiver by direct transmission. When SU in DT, the achievable rate from ST i to SR i by using PT j 's channel is $R_{i,j,D} = (W_j/3) \log_2(1 + (P_i|G_{ST_i}^{SR_i}|^2/\sigma^2))$. It is observed that the SU can always achieve a larger utility when it works in the proposed cooperative way, regardless of its transmit power (Here we only show the utilities of SU2 and SU5, and other SUs' performance can be analyzed in a similar manner). With such a large power supply, the SU can get more power at a lower price, thus receiving larger utility in CT method than in DT method.

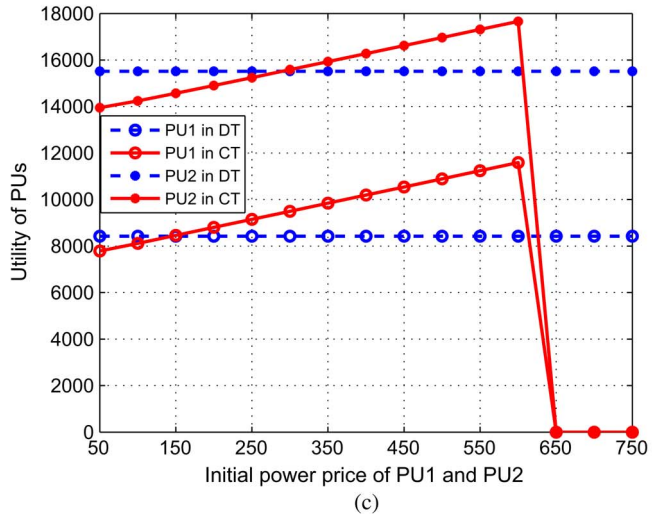
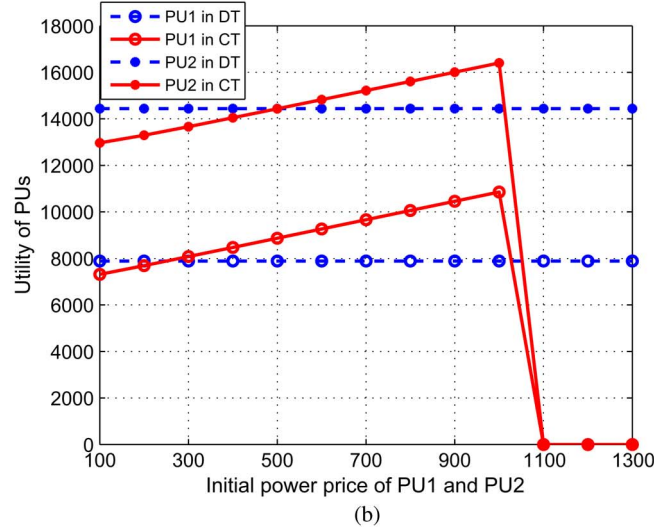
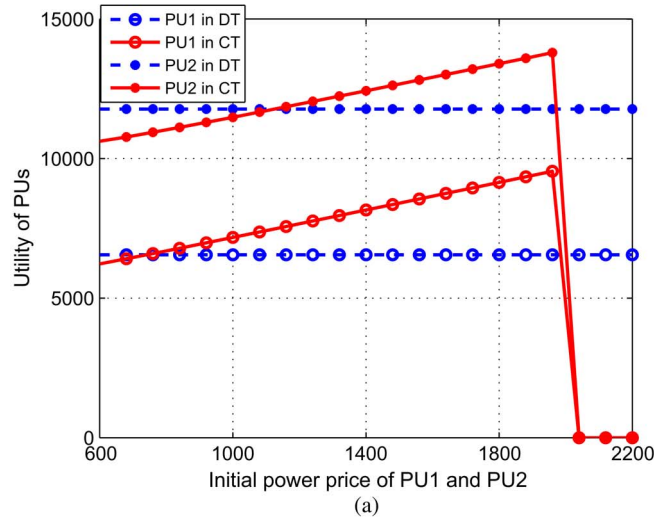


Fig. 10. Impacts of initial price on the utility achieved in CT. (a) Power supplies: 1 W. (b) Power supplies: 4 W. (c) Power supplies: 7 W.

VI. CONCLUSION

In this article, we tackled the joint power and spectrum allocation problem under a new cooperative CR framework, where primary users assist secondary transmissions and earn revenue

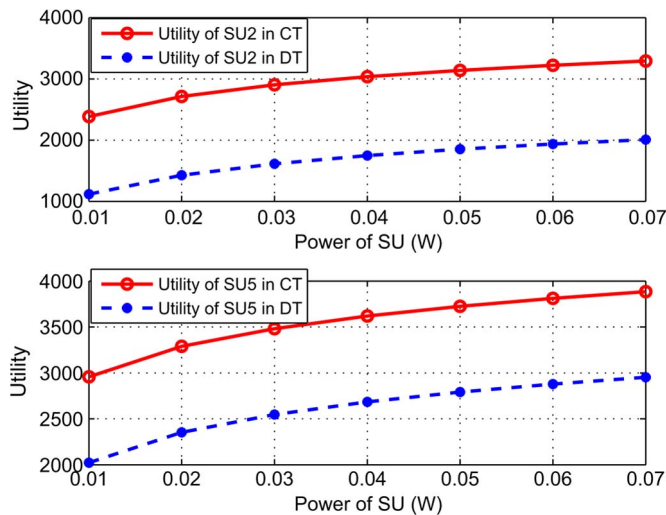


Fig. 11. Cooperative transmission vs. direct transmission for the SU.

from selling the spectrum and cooperative power to secondary users. The trade between primary users and secondary users is modeled as an auction with two bundling commodities. The auction algorithm and its convergence performance are investigated. Future work can be extended to the cases in which the spectrum can sell either separately or together with the cooperative power. In this way, secondary users can choose cooperative transmission (buy the spectrum and power both) or direct transmission (buy the spectrum only) for maximizing the utility.

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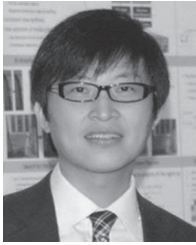


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